B.A./B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH6DSE31

(Mathematical Modelling)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) What are the limitations of mathematical modelling?

(b) Write down the assumptions of queueing model $(M/M/1):(N/FCFS/\infty)$.

(c) Find the average length of non-empty queue of a system $(M/M/1):(\infty/FCFS/\infty)$.

(d) Write down the relations between

(i) L_s and L_q

(ii) W_s and W_q of $(M/M/1):(\infty/FCFS/\infty)$

(e) What do you mean by service discipline of a queueing system?

(f) What is Allee effect?

(g) What is Malthns model?

(h) Define the Lotka-Volterra model for prey-predator system.

(i) Give an example of two species competition model.

(j) Define equilibrium point of a system.

(k) Give an example of a discrete prey-predator model.

(l) Find the equilibrium point of $\frac{dx}{dt} = x(x-1)$.

(m) Write down the logistic model of population growth explaining the different terms involved in it.

(n) What are the state variables for the dynamical models of ecosystem?

(o) What are the basic postulates for developing continuous time models of single species population?

5×4=20

2. Answer *any four* questions:

(a) A population satisfies the growth equation $x_{n+2} - 2x_{n+1} + 3x_n = 0$. Find the population in n-th generation. Also, find the steady state.

(b) Write down the logistic model for a single-species population. Hence, explain the concepts of carrying capacity and intra-species competition. 2+3

(c) Discuss density dependent growth model.

(d) Find the non-negative equilibrium of a population governed by $x_{n+1} = \frac{2x_n^2}{x_n^2+2}$ and investigate the stability. 3+2

(e) If the arrival process in a queueing system follows the poisson distribution then show that the associated random variable defined as inter-arrival time follows the exponential distribution.

(f) Discuss different states of a queueing system.

10×2=20

3. Answer *any two* questions:

(a) Obtain the maximum likelihood estimator of σ^2 where μ (known) and σ are mean and standard deviation of a normal population respectively. Show that this estimator is unbiased. 8+2

(b) In a railway yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes, calculate 2×5

(i) the average number of trains in the system.

(ii) the average number of trains in the queue.

(iii) the expected waiting time in the system.

(iv) the expected waiting time in the queue.

(v) the probability that the number of trains in the system exceeds 10.

(c) Consider the prey-predator system

$$\frac{dx}{dt} = x(1 - x - y)$$

$$\frac{dy}{dt} = \beta(x - \alpha)y, \quad \alpha, \beta \text{ being constants.}$$

Investigate the nature of equilibrium points of the system.

(d) Define a cooperative system and give an example. Prove that the orbit of a system that is cooperative either converge to equilibrium or diverge to infinity.

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